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Robust control design for multivariable plants with time-delays

Dominique Nelson Gruel*, Patrick Lanusse, Alain Oustaloup

IMS - CNRS UMR 5218 - Universite de Bordeaux - ENSEIRB, LAPS Department, 351 cours de la Liberation Bat A, 33405 Talence, France

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ABSTRACT

This paper proposes an extension of CRONE control and design for unstable, multiple time-delay multivariable plants with right half-plane (RHP) zeros. The CRONE control approach developed for multivariable square plants is based on third generation scalar CRONE methodology. The aim is to find a decoupling and stabilizing controller for the open-loop transfer matrix. Fractional order transfer functions are used to define all the components of the diagonal open-loop transfer matrix, β_{0i}. Optimization gives the best fractional open-loop transfer matrix, β. Finally, frequency-domain system identification is used to find a robust controller $K = G^{-1}\beta$. When the plant is inversed, time-delays and RHP poles or zeros can appear on the denominator of the transfer matrix, G^{-1}. To achieve a stable controller some time-delays, RHP zeros and unstable poles must be included within the diagonal transfer function of the open-loop transfer matrix. To assess the proposed design, the CRONE control approach is applied to a distillation example.
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1. Introduction

There are many types of robustness, but our proposed approach focuses on the stability degree [1–8]. The aim of the multivariable (MIMO) CRONE control approach is to robustify closed-loop dynamic performance through robustness of the damping factor or the resonant peaks of control, when plant parameters vary. Contrary to some methods, CRONE control and design does not deal with robustness of the closed-loop bandwidth and thus, permits a limiting of the control effort.

The CRONE control system design is based on the common unity-feedback configuration (Fig. 1). The CRONE methodology uses integro-differentiation with non-integer, or fractional, orders to define the optimal controller or open-loop. Three generations of CRONE control have been developed, successively extending the application field.

The first generation based on differentiation with real fractional orders proposes a controller with no variation on phase around the open-loop gain crossover frequency ω_{cg}. This kind of controller provides a robust phase margin for plants with a constant phase, often found in the high frequencies. Such a controller can lead to high levels of control input.

Second generation CRONE control proposes a fractional open-loop transfer function with no variation on phase around the gain crossover frequency ω_{cg}. The open-loop Nichols locus around ω_{cg} is a vertical straight line which ensures the robustness of phase and modulus margins and of resonant peaks of complementary sensitivity and sensitivity functions.

Finally, the third generation based on differentiation with complex fractional orders must be used when the plant frequency uncertainty domains are of various types (not only gain-like). The vertical template is then replaced by a generalized template. It is always described as a straight line in the Nichols chart, but of any direction. The direction is provided by complex fractional order integration, or by a curvilinear template defined by a set of generalized templates.

CRONE control design has been extended to stable and minimum phase square MIMO plants. Now, a totally multivariable approach is proposed for uncertain and unstable square $n \times n$ MIMO plants with right half-plane (RHP) zeros and time-delays. Section 2 presents differentiation with fractional orders. Sections 3 and 4 present CRONE control of multivariable plants with time-delay. Simulation is presented in Section 5.

2. Fractional integro-differentiation

Cauchy's integral formula is given by

$$f(t) = \frac{1}{2\pi i}\int_C \frac{f(\tau)}{\tau - t} d\tau, \tag{2.1}$$

with

- $f : U \to \mathbb{C}$ a holomorphic function,
- U a subset of $\mathbb{C}$,

* Corresponding author. Tel.: +33 540 003 627; fax: +33 540 006 644.
E-mail address: dominique.nelson@laps.ims-bordeaux.fr (D.N. Gruel).

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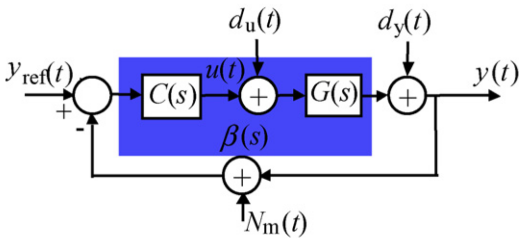

Fig. 1. Common CRONE control system diagram.

- C border circle of D,
- $D=\{\tau/|\tau-t|\leq\varepsilon\}\subset U$,

Sonin and Letnikov in [9] and [10] gave the first definition of differentiation through the υth derivative of (2.1):

$$D^{\upsilon}f(t)=\frac{\upsilon!}{2\pi i}\int_C\frac{f(\tau)}{(\tau-t)^{\upsilon+1}}d\tau. \tag{2.2}$$

(2.2) is stretched out to non-integer orders by extending $\upsilon!$ to arbitrary values (since $\upsilon!=\Gamma(\upsilon+1)$).

This formalism is limited to negative real parts of the differentiation order. The extension of integro-differentiation orders to fractional complex sets ($\Re e(n_f)>0$) goes back to the nineteenth century and the work of Liouville and Riemann [11–13]. The Riemann–Liouville fractional integral ($\Re e(n_f)>0$) is expressed as

$$I_{t_0}^{n_f}f(t)=\frac{1}{\Gamma(n_f)}\int_{t_0}^{t}\frac{f(\tau)}{(t-\tau)^{1-n_f}}d\tau, \tag{2.3}$$

with

- $f(t)\in L_1(t_0,t)$,
- $t>t_0$,
- $t_0\in\mathbb{R}$,
- $n_f\in\mathbb{C}$,
- $\Gamma(n_f)$ the Gamma function extended to complex sets:

$$\Gamma(n_f)=\int_0^{\infty}e^{-x}x^{n_f-1}\,dx. \tag{2.4}$$

Eq. (2.3) can be interpreted as the convolution between $f(t)$ and the function $h(t)=(t^{n_f-1}/\Gamma(n_f))u(t)$. In fact:

$$I_0^{n_f}f(t)=\frac{1}{\Gamma(n_f)}\int_0^{t}\frac{f(\tau)}{(t-\tau)^{1-n_f}}\,d\tau=h(t)\otimes f(t). \tag{2.5}$$

Consequently, the Laplace transform of (2.5) gives:

$$\begin{aligned}L\left\{I_0^{n_f}f(t)\right\}&=L\left\{h(t)\otimes f(t)\right\}\\&=L\left\{\frac{t^{n_f-1}u(t)}{\Gamma(n_f)}\right\}L\left\{f(t)\right\}=\frac{1}{s^n}L\left\{f(t)\right\}.\end{aligned} \tag{2.6}$$

The fractional order of the Riemann–Liouville fractional derivative ($\Re e(n_f)>0$) is

$$n_f=\left\lfloor\Re e(n_f)\right\rfloor+\left\{\Re e(n_f)\right\}+i\ Im(n_f), \tag{2.7}$$

where

- $n_f\in\mathbb{C}$,
- $\left\lfloor\Re e(n_f)\right\rfloor$ the integer part of n_f,
- $\{\Re e(n_f)\}$ the fractional part of n_f, $0\leq n_f<1$.

n_f can also be written:

$$n_f=\left\lfloor\Re e(n_f)\right\rfloor+1-(1-\{\Re e(n_f)\})+i\ Im(n_f), \tag{2.8}$$

with

- $m_f=\left\lfloor\Re e(n_f)\right\rfloor+1$,
- $n'_f=-(1-\{\Re e(n_f)\})+i\,Im(n_f)$,

and thus

$$n_f=m_f+n'_f. \tag{2.9}$$

When $\Re e(n_f)<0$, the fractional integration becomes a $-n_f$ order fractional derivative:

$$I_{t_0}^{n_f}f(t)=D_{t_0}^{-n_f}f(t), \tag{2.10}$$

and reciprocally:

$$D_{t_0}^{n_f}f(t)=I_{t_0}^{-n_f}f(t). \tag{2.11}$$

Consequently, the Riemann–Liouville fractional derivative is defined by the integer derivative of the fractional integration:

$$D_{t_0}^{n_f}f(t)=\frac{d^{m_f}}{dt}(I_{t_0}^{-n'_f}f(t)). \tag{2.12}$$

From (2.9), n'_f can be written:

$$n'_f=m_f-n_f, \tag{2.13}$$

and (2.12) becomes:

$$D_{t_0}^{n_f}f(t)=\frac{1}{\Gamma(m_f-n_f)}\times\frac{d^{m_f}}{dt}\left(\int_0^t\frac{f(\tau)}{(t-\tau)^{1-(m_f-n_f)}}d\tau\right). \tag{2.14}$$

This relation is defined for every $f(t)$ such that

$$f(t)=(t-t_0)^{\lambda}\varphi(t-t_0), \tag{2.15}$$

or

$$f(t)=(t-t_0)^{\lambda}\ln(t-t_0)\varphi(t-t_0), \tag{2.16}$$

with

- $\lambda\in\mathbb{C}$,
- $\Re e(n_f)>-1$,
- $\varphi(t)$ analytic function of $\mathbb{C}$ for $t>0$.

The Laplace transform of (2.14) gives, with null initial conditions:

$$L\{D_0^{n_f}f(t)\}=L\{D_0^{m_f}(I_0^{m_f-n_f}f(t))\}=s^{m_f}L\{I_0^{m_f-n_f}f(t)\}. \tag{2.17}$$

Using (2.6), (2.17) becomes:

$$L\{D_0^{n_f}f(t)\}=s^{m_f}s^{n_f-m_f}L\{f(t)\}=s^{n_f}L\{f(t)\}. \tag{2.18}$$

These final definitions of generalized integro-differentiation reveal that the fractional derivative or integral of a causal function at given time takes into account the whole past of the function. Also the relation obtained by the Laplace transform of (2.5) and (2.14) extends the integer results. From 1975 on, Oustaloup et al. has proposed various methodologies:

- for synthesizing band-limited differentiators whose orders are real or complex and fractional [14];
- for designing real non-integer order robust controllers [4].

3. CRONE control methodology for MIMO systems

The aim of this methodology is to find a diagonal open-loop transfer matrix with n fractional order transfer elements [7,15]. This transfer matrix is parametered to satisfy the four following objectives:

- perfect decoupling for the nominal plant,
- accuracy specifications at low frequencies,
- required nominal stability margins of the closed-loops (behaviours around the required cut-off frequencies),
- specifications on the n control efforts at high frequencies.

After an optimization (minimizes a robustness cost) of the various parameter of the open-loop transfer matrix, frequency-domain system identification is used to obtain the fractional controller.

Our aim is to achieve output feedback decoupling. Thus, the decoupling and diagonal open-loop transfer matrix will permit a nominal closed-loop transfer matrix to be diagonal:

$$\beta_0(s)=\begin{bmatrix}\beta_{0_1}(s) & 0 & & \cdots & 0\\ 0 & \ddots & & & \\ & & \beta_{0i}(s) & & \cdots\\ \vdots & & & & \\ & & & \ddots & 0\\ 0 & & \cdots & 0 & \beta_{0n}(s)\end{bmatrix}. \tag{3.1}$$

The nominal sensitivity, complementary sensitivity, input sensitivity and input-disturbance sensitivity function transfer matrices are

$$S_0(s)=[I+\beta_0(s)]^{-1}, \tag{3.2}$$

$$T_0(s)=[I+\beta_0(s)]^{-1}\beta_0(s), \tag{3.3}$$

$$S_{U0}(s)=K(s)[I+\beta_0(s)]^{-1}=K(s)S_0(s), \tag{3.4}$$

$$S_{i0}(s)=[I+\beta_0(s)]^{-1}G(s)=T_0(s)K^{-1}(s), \tag{3.5}$$

(3.2) and (3.3) are written:

$$T_0(s)=diag[T_{0i}(s)]_{1\le i\le n}, \tag{3.6}$$

and

$$S_0(s)=diag[S_{0i}(s)]_{1\le i\le n}, \tag{3.7}$$

with

$$T_{0i}(s)=\frac{\beta_{0_i}(s)}{1+\beta_{0_i}(s)}, \tag{3.8}$$

and

$$S_{0i}(s)=\frac{1}{(1+\beta_{0_i}(s))}. \tag{3.9}$$

The open-loop transfer functions $\beta_{0_i}(s)$ as defined in the following section are used to satisfy the three other objectives.

3.1. Definition of the diagonal open-loop transfer function elements

The open-loop transfer function behaviour can be described by using the third generation CRONE control methodology presented in this section. As mentioned above, the use of real fractional order integration on frequency range $[\omega_A,\omega_B]$ produces a straight line on the Nichols chart which is called the *generalized template* (Fig. 2).

The generalized template can be defined by an integrator of complex fractional order $n_f = a + ib$ whose real part determines its phase location at frequency ω_{cg}, that is $-\Re e(n_f)\pi/2$, and whose imaginary part determines its angle to the vertical. It is described by the limitation in the operational plane $\mathbb{C}_j$ of the complex non-integer integrator transfer function:

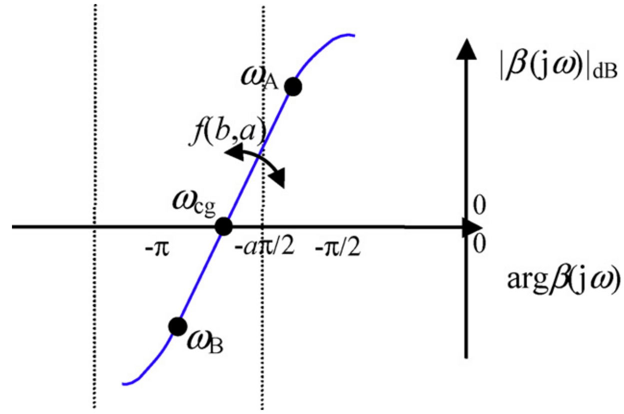

Fig. 2. Generalized template on the Nichols plane.

$$\beta_{0_i}(s)=\left[\left(\frac{\omega_{cg}}{s}\right)^{n_f}\right]_{C_j}, \tag{3.10}$$

with

- $s=\sigma+j\omega\in\mathbb{C}_j$,
- $n_f = a + ib \in \mathbb{C}_i$,

also written as

$$\beta_{0_i}(s)=\left(\cosh\left(b\frac{\pi}{2}\right)\right)^{\mathrm{sign}(b)}\left(\frac{\omega_{cg}}{s}\right)^{a}\times\left(\Re e_{/i}\left(\left(\frac{\omega_{cg}}{s}\right)^{ib}\right)\right)^{-\mathrm{sign}(b)}. \tag{3.11}$$

This transfer function can be described as based on band-limited complex non-integer integration:

$$\beta_{0_i}(s)=C^{\mathrm{sign}(b)}\left(\frac{1+s/\omega_h}{1+s/\omega_l}\right)^{a}\times\left(\Re e_{/i}\left\{\left(C_g\frac{1+s/\omega_h}{1+s/\omega_l}\right)^{ib}\right\}\right)^{-q\,\mathrm{sign}(b)}, \tag{3.12}$$

with

$$C=ch\left[b\left(\arctan\left(\frac{\omega_{cg}}{\omega_l}\right)-\arctan\left(\frac{\omega_{cg}}{\omega_h}\right)\right)\right], \tag{3.13}$$

and

$$C_g=\left(\frac{1+(\omega_{cg}/\omega_l)^2}{1+(\omega_{cg}/\omega_h)^2}\right)^{1/2}. \tag{3.14}$$

The corner frequencies are placed around the extreme frequencies ω_A and ω_B such that

$$\omega_l<\omega_A<\omega_{cg}<\omega_B<\omega_h. \tag{3.15}$$

For stable and minimum phase plants, the generalized template is taken into account in the open-loop transfer function as follows:

$$\beta_{0_i}(s)=\beta_{l_i}(s)\beta_{0_i}(s)\beta_{h_i}(s), \tag{3.16}$$

with

$$\beta_{l_i}(s) = C_{l_i}\left(\frac{\omega_{l_i}}{s}+1\right)^{n_{l_i}}, \tag{3.17}$$

where order n_{l_i} fixes the accuracy of each closed-loop,

$$\beta_{h_i}(s) = \frac{C_{h_i}}{((s/\omega_{h_i})+1)^{n_{h_i}}}, \tag{3.18}$$

and order n_{h_i} permits the elements of the controller to be proper.

This third generation CRONE control open-loop transfer function has thus been defined using the gain crossover open-loop frequency. However, this definition can also be made using the closed-loop resonance frequency where ω_r replaces ω_{cg}.

3.2. Decoupling and optimized controller

Let G_0 be the nominal plant transfer matrix such that $G_0(s) = [g_{0_{ij}}(s)]_{i,j\in N}$ ($g_{0_{ij}}(s)$ is a strictly proper transfer function) and:

$$\beta_0 = G_0K = diag[\beta_{0_i}] = diag\left[\frac{n_i}{d_i}\right]_{i\in N}, \tag{3.19}$$

where

- $N=\{1, \ldots, n\}$,
- $\beta_{0_i} = n_i/d_i$ the element of the ith column and row.

As mentioned above the aim of CRONE control for MIMO plants is to find a decoupling controller for the nominal plant. G_0 being not diagonal, the problem is to find a decoupling and stabilizing controller K [16]. This controller exists iff the following hypotheses are verified:

$$H_1 : [G(s)]^{-1}\text{exists}, \tag{3.20}$$

$$H_2 : Z_+[G(s)] \cap P_+[G(s)] = 0, \tag{3.21}$$

where $Z_+[G(s)]$ and $P_+[G(s)]$ indicate the positive real part zero and pole sets.

The controller $K(s)$ is given by

$$K = G_0^{-1}\beta_0 = \frac{adj(G_0)}{|G_0|}diag\left[\frac{n_i}{d_i}\right]_{i\in N} \tag{3.22}$$

with

- $adj(G_0(s)) = [G_0^{ij}(s)]^{\mathrm{T}} = [G_0^{ji}(s)]$,
- $G_0^{ij}(s)$ the cofactor of element $g_{0_{ij}}(s)$,
- $|G_0|$ the determinant of $G_0(s)$,

and thus

$$k_{ij} = \frac{G_0^{ji}}{|G_0|}\beta_{0_i} \quad \forall i,j\in N. \tag{3.23}$$

For plants other than the nominal, the closed-loop transfer matrices $T(s)$ and $S(s)$ are not diagonal anymore. Each diagonal element $T_{ii}(s)$ and $S_{ii}(s)$ could be interpreted as a closed-loop transfer function resulting from a scalar open-loop transfer function $\beta_{ii}(s)$ called *equivalent open-loop transfer function* [17]:

$$\beta_{ii}(s) = \frac{T_{ii}(s)}{1-T_{ii}(s)} = \frac{1-S_{ii}(s)}{S_{ii}(s)}. \tag{3.24}$$

For each nominal open-loop β_{0_i} various generalized templates can tangent the same required magnitude-contour of the Nichols chart or the same resonant peak $M_{p_{0_i}}$. The optimal template is the one that best minimizes the robustness cost function:

$$J = \sum_{i=1}^{n}(M_{p_{\max_i}} - M_{p_{\min_i}})^2, \tag{3.25}$$

where

$$M_{p_{\max_i}} = \max_G\sup_\omega(T_{ii}(j\omega)) = \max_G\sup_\omega\left(\frac{\beta_{ii}(j\omega)}{1+\beta_{ii}(j\omega)}\right), \tag{3.26}$$

$$M_{p_{\min_i}} = \min_G\sup_\omega(T_{ii}(j\omega)) = \min_G\sup_\omega\left(\frac{\beta_{ii}(j\omega)}{1+\beta_{ii}(j\omega)}\right), \tag{3.27}$$

while respecting the following set of inequalities for $\omega\in\mathbb{R}$ and $i, j\in N$:

$$\inf_G\left|T_{ij}(j\omega)\right| \geq T_{ij_l}(\omega), \tag{3.28}$$

$$\sup_G\left|T_{ij}(j\omega)\right| \leq T_{ij_u}(\omega), \tag{3.29}$$

$$\sup_G\left|S_{ij}(j\omega)\right| \leq S_{ij_u}(\omega), \tag{3.30}$$

$$\sup_G\left|KS_{ij}(j\omega)\right| \leq KS_{ij_u}(\omega), \tag{3.31}$$

$$\sup_G\left|SG_{ij}(j\omega)\right| \leq SG_{ij_u}(\omega), \tag{3.32}$$

where G is the nominal or perturbed plant.

As the uncertainties are taken into account by the least conservative method, a non-linear optimization method must be used to find the optimal values of the independent parameters of the fractional open-loop, and consequently to find an optimal placement of the equivalent open-loop frequency response $\beta_{ii}(j\omega)$.

The complex order is a tuning parameter that has the advantage of replacing a whole set of parameters found in common rational controllers.

4. CRONE control design for unstable MIMO plants with RHP zeros and multiple time-delays

Let the nominal plant transfer matrix G_0 be

$$G_0(s) = \begin{bmatrix} g_{0_{11}}(s) & & \cdots & g_{0_{1n}}(s) \\ & \ddots & & \\ & & g_{0_{ij}}(s) & \vdots \\ \vdots & & & \\ & & \ddots & \\ g_{0_{n1}}(s) & \cdots & & g_{0_{nn}}(s) \end{bmatrix}, \tag{4.1}$$

where

- $g_{0_{ij}}(s) = h_{ij}(s)e^{-L_{ij}s}$,
- $h_{ij}(s)$ is a strictly proper time-delay free transfer function,
- L_{ij} is a positive constant.

Considering the general case where G_0 has RHP zeros and time-delays, the inverse of G_0 is written:

$$P(s) = G_0^{-1}(s) = \frac{\left[G_0^{ij}(s)\right]^{\mathrm{T}}}{\det(G_0(s))} = \frac{\left[G_0^{ji}(s)\right]}{\det(G_0(s))}, \tag{4.2}$$

where

$$\det(G_0(s)) = \frac{\sum_{k=0}^{q_1} n_k(s)e^{-\alpha_k s}}{b_0(s)}, \tag{4.3}$$

each G_0^{ji} can be written:

$$\frac{\sum_{l=0}^{q_{2ji}} m_{l_{ji}}(s)e^{-\delta_{l_{ji}} s}}{d_{0_{ji}}(s)}, \tag{4.4}$$

with

- $n_k(s)$, $m_{l_{ji}}(s)$, $b_0(s)$ and $d_{0_{ji}}(s)$ non-zero scalar polynomials of s,
- $\alpha_0 < \alpha_1 < \ldots < \alpha_{q_1}$,
- $\delta_{0_{ji}} < \delta_{1_{ji}} < \ldots < \delta_{q_{2_{ji}}}$.

We define the delay of the non-zero transfer function $a(s)$ by $\tau(a(s))$ which is the smallest time-delay of $a(s)$. As a consequence, $\tau(a(s))$ cannot be negative. It is easy to verify:

- $\tau(a_1 a_2) = \tau(a_1) + \tau(a_2)$,
- $\tau(a^{-1}) = -\tau(a)$,

$\forall\, a,\ a_1$ and a_2 are non-zero transfer functions.(4.2) becomes:

$$P(s) = G_0^{-1}(s) = \begin{bmatrix} p_{11}(s)e^{\gamma_{11}s} & \cdots & p_{1n}(s)e^{\gamma_{1n}s} \\ \vdots & p_{ij}(s)e^{\gamma_{ij}s} & \vdots \\ p_{n1}(s)e^{\gamma_{n1}s} & \cdots & p_{nn}(s)e^{\gamma_{nn}s} \end{bmatrix}, \tag{4.5}$$

where

- $p_{ij}(s) = f(n_k, m_{l_{ji}}, b_0, d_{0_{ji}}, \delta_{l_{ji}}, \alpha_k), \quad k \in [1, q_1]$ and $l \in [1, q_{2_{ji}}]$,
- $\gamma_{ij} = f(\alpha_0, \delta_{0_{ji}})$.

Consequently, the controller is written:

$$k_{ij}(s) = p_{ij}(s)e^{\gamma_{ij}s}\beta_{0_i}(s). \tag{4.6}$$

The relation above, implies that time-delay, RHP zeros and unstable poles of p_{ij} must appear in β_{0_i} to make the controller achievable and stable. The open-loop transfer matrix will now be defined. To treat this type of plant (unstable poles, RHP zeros and time-delays) the method consisted in:

- find the time-delay of all components of the open-loop transfer function $\beta_{0_i}(s)$;
- -find the positive real part zeros and unstable poles of p_{ij}, that must appear on the open-loop transfer function matrix.

4.1. First step: time-delay

The following condition must be verified:

$$\tau(k_{ij}) \geq 0 \quad \forall\, i, j \in N, \tag{4.7}$$

For this to be true, with (4.6):

$$\tau(\beta_{0_i}(s)) \geq \gamma_{ij}, \tag{4.8}$$

where

$$\gamma_{ij} = \tau(|G_0|) - \tau(G_0^{ji}) = \alpha_0 - \delta_{0_{ji}}. \tag{4.9}$$

The time-delay of the ith open-loop transfer function must satisfy all the following relations:

$$\begin{aligned} &\tau\left(\beta_{0_i}\right) \geq \tau\left(\left|G_0\right|\right) - \tau\left(G_0^{1i}\right) \\ &\tau\left(\beta_{0_i}\right) \geq \tau\left(\left|G_0\right|\right) - \tau\left(G_0^{2i}\right) \\ &\vdots \\ &\tau\left(\beta_{0_i}\right) \geq \tau\left(\left|G_0\right|\right) - \tau\left(G_0^{ni}\right) \end{aligned} \tag{4.10}$$

Finally:

$$\tau(\beta_{0_i}) \geq \tau(|G_0|) - \tau_i \quad \forall\, i \in N, \tag{4.11}$$

with

$$\tau_i = \min_{j \in N}(\tau(G_0^{ji})) \tag{4.12}$$

Relation (4.11) implies that the ith open-loop transfer function must have a time-delay higher or equal to the difference between the time-delay of $|G_0|$ and, considering one column, the minimum time-delay of $[G_0^{ji}]$.

4.2. Step two: RHP poles and zeros

Let the transfer function $a(s)$ be

$$a(s) = \frac{Z_p^+ Z_p^-}{P_p^+ P_p^-}, \tag{4.13}$$

where

- Z_p^+ is the positive real part zero set: $Z_p^+ = \left\{z \in C^+;\ a(z) = 0\right\}$,
- P_p^+ is the positive real part pole set: $P_p^+ = \left\{p \in C^+;\ a^{-1}(p) = 0\right\}$,
- Z_p^- is the negative real part zero set: $Z_p^- = \left\{z \in C^-;\ a(z) = 0\right\}$,
- P_p^- is the negative real part pole set: $P_p^- = \left\{p \in C^-;\ a^{-1}(p) = 0\right\}$.

Let $\eta_z(a)$ be integer ξ such that $\lim_{s \to z} \frac{a(s)}{(s-z)^\xi}$ exists and is non-zero. So for a given z:

- $a(s)$ has a $\eta_z(a)$ order zero at z if $\eta_z(a) > 0$,
- $a(s)$ has a $\eta_z(a)$ order pole at z if $\eta_z(a) < 0$,
- $a(s)$ has neither pole nor zero if $\eta_z(a) = 0$.

It is easy to verify that:

- $\eta_z(a_1 a_2) = \eta_z(a_1) + \eta_z(a_2)$,
- $\eta_z(a^{-1}) = -\eta_z(a)$.

$\forall\, a,\ a_1$ and a_2 are non-zero transfer functions.

A transfer function is stable and minimum phase iff [18,19]:

$$\eta_z(a) = 0 \quad \forall\, z \in \mathbb{C}^+. \tag{4.14}$$

In our case we need:

$$\eta_z(k_{ij}) \geq 0 \quad \forall\, i, j \in N \quad \forall\, z \in \mathbb{C}^+, \tag{4.15}$$

as, contrary to single-input/single-output systems, the controller of MIMO systems can have RHP zeros. The stability of transfer KS and TK^{-1} depends only on the stability of K.

With (4.6), (4.15) becomes:

$$\eta_z(k_{ij}(s)) \geq \eta_z(p_{ij}(s)\beta_{0_i}(s)), \tag{4.16}$$

Let $\left|G_0\right|_{\not{\alpha}_0}$ and $\left[G_0^{ji}\right]_{\not{\delta}_{0ji}}$ be the transfer functions that appear in the factorized form of

$$\left|G_0\right| = \left|G_0\right|_{\not{\alpha}_0} e^{-\alpha_0}, \tag{4.17}$$

$$\left[G_0^{ji}\right] = \left[G_0^{ji}\right]_{\delta_{0_{ji}}} e^{-\delta_{0_{ji}}}, \quad (4.18)$$

with

$$\left|G_0\right|_{\alpha_0} = \frac{n_0 + \sum_{k=1}^{q_1} n_k(s) e^{-(\alpha_k - \alpha_0)s}}{b_0(s)}, \quad (4.19)$$

$$\left[G_0^{ji}\right]_{\delta_{0_{ji}}} = \frac{m_{0_{ji}} + \sum_{l=1}^{q_{2_{ji}}} m_{l_{ji}}(s) e^{-(\delta_{l_{ji}} - \delta_{0_{ji}})s}}{d_{0_{ji}}(s)}, \quad (4.20)$$

With (4.17) and (4.18), (4.16) becomes:

$$\eta_z(k_{ij}) \geq \eta_z\left(\frac{\left[G_0^{ji}\right]_{\delta_{0_{ji}}}}{\left|G_0\right|_{\alpha_0}} \beta_{0_i}\right), \quad (4.21)$$

Consequently,

$$\eta_z(\beta_{0_i}) \geq \eta_z\left(\left|G_0\right|_{\alpha_0}\right) - \eta_z\left(\left[G_0^{ji}\right]_{\delta_{0_{ji}}}\right). \quad (4.22)$$

The ith open-loop transfer function must satisfy all the following equations:

$$\begin{array}{l} \eta_z(\beta_{0_i}) \geq \eta_z\left(\left|G_0\right|_{\alpha_0}\right) - \eta_z\left(\left[G_0^{1i}\right]_{\delta_{01i}}\right) \\ \eta_z(\beta_{0_i}) \geq \eta_z\left(\left|G_0\right|_{\alpha_0}\right) - \eta_z\left(\left[G_0^{2i}\right]_{\delta_{02i}}\right) \\ \quad \vdots \qquad\qquad \vdots \\ \eta_z(\beta_{0_i}) \geq \eta_z\left(\left|G_0\right|_{\alpha_0}\right) - \eta_z\left(\left[G_0^{ni}\right]_{\delta_{0ni}}\right) \end{array} \quad (4.23)$$

Finally:

$$\eta_z(\beta_{0_i}) \geq \eta_z\left(\left|G_0\right|_{\alpha_0}\right) - \eta_i(z) \ \forall i \in N, \quad (4.24)$$

with

$$\eta_i(z) = \min_{j \in N}\left(\eta_z\left([G_0^{ji}]_{\delta_{0_{ji}}}\right)\right). \quad (4.25)$$

Eq. (4.24) also implies that the ith open-loop transfer function $\beta_{0_i}(s)$ must have, for each z, a $(s-z)$ transfer function of order $\eta z(\beta_{0_i})$. $\eta z(\beta_{0_i})$ is higher or equal to the difference between the order of z in $\left|G_0\right|_{\alpha_0}$ and, considering one column, is the common order of z in $[G_0^{ji}]_{\delta_{0_{ji}}}$. The sign of $\eta z(\beta_{0_i})$ determines if the transfer function is a RHP zero or an unstable pole of $\beta_{0_i}(s)$.

Finally, when all zeros and poles that must be integrated in the ith open-loop transfer function are found, $\beta_{0_i}(s)$ is

$$\beta_{0_i}(s) = \beta_{z_i}(s)\beta_{p_i}(s)\beta_{l_i}(s)\beta_{0_i}(s)\beta_{h_i}(s), \quad (4.26)$$

with

$$\beta_{z_i} = C_{z_i} \Pi k = 1 N_{z_i} (z_{i_k} - s)^{\eta z_{i_k}(\beta_{0_i})}, \quad (4.27)$$

$$\beta_{p_i} = C_{p_i} (e^{-j\pi})^{\Psi_{p_i}} \Pi k = 1 N_{p_i} \left(\frac{p_{i_k} + s}{p_{i_k} - s}\right)^{\eta p_{i_k}(\beta_{0_i})}, \quad (4.28)$$

where

- z_{i_k} is a z such that $\eta z(\beta_{0_i}) > 0$,
- p_{i_k} is a z such that $\eta z(\beta_{0_i}) < 0$,
- $\eta z_{i_k}(\beta_{0_i}) = \eta z(\beta_{0_i})$, if z is a zero of $\beta_{0_i}(s)$,
- $\eta p_{i_k}(\beta_{0_i}) = -\eta z(\beta_{0_i})$, if z is a pole of $\beta_{0_i}(s)$,
- N_{z_i} is the number of RHP zeros of $\beta_{0_i}(s)$,
- N_{p_i} is the number of RHP poles of $\beta_{0_i}(s)$,
- $\Psi_{p_i} = \sum_{k=1}^{N_{p_i}} \eta p_{i_k}(\beta_{0_i})$,
- $\Psi_{z_i} = \sum_{k=1}^{N_{z_i}} \eta z_{i_k}(\beta_{0_i})$.

In addition, the controller must be proper and must permit the rejection of low disturbance.

The controller is written:

$$K(s) = G_0^{-1}(s)\beta(s). \quad (4.29)$$

With β a diagonal transfer matrix, consequently:

$$k_{ij}(s) = P_{ij}(s)\beta_{0_i}(s). \quad (4.30)$$

The controller is proper iff:

$$deg(k_{ij}(s)) \geq 0. \quad (4.31)$$

$$(deg(N_{p_{ij}(s)}) - deg(D_{p_{ij}(s)})) + ((deg(n_i(s)) - deg(d_i(s)))) \geq 0, \quad (4.32)$$

finally

$$deg(d_i(s)) + deg(D_{p_{ij}(s)}) \geq deg(N_{p_{ij}(s)}) + deg(n_i(s)) \quad \forall j \in N, \quad (4.33)$$

with

- $p_{ij}(s) = \frac{N_{p_{ij}(s)}}{D_{p_{ij}(s)}}$,
- $\beta_{0_i}(s) = \frac{n_i(s)}{d_i(s)}$.

As a consequence:

$$deg(d_i(s)) - deg(n_i(s)) \geq \max_{j \in N}(deg(N_{p_{ij}(s)}) - deg(D_{p_{ij}(s)})). \quad (4.1)$$

At high frequencies:

$$deg(d_i(s)) = n_{h_i}, \quad (4.2)$$

$$deg(n_i(s)) = \Psi_{z_i}, \quad (4.3)$$

finally

$$n_{h_i} \geq \Psi_{z_i} + \max_{j \in N}(deg(N_{p_{ij}(s)}) - deg(D_{p_{ij}(s)})). \quad (4.37)$$

At low frequencies, disturbances are rejected if:

$$\eta_{z=0}(S(s)) + \eta_{z=0}(G_0(s)) \geq 0. \quad (4.4)$$

At low frequencies $S(s)$ is equivalent to $\beta_{0_i}(s)$, consequently:

$$\eta_{z=0}(S(s)) \approx \eta_{z=0}(\beta_{0_i}(s)) = n_{l_i} + \Psi_{z_i}, \quad (4.5)$$

and

$$\eta_{z=0}(G_0(s)) = \eta_{z=0}(N_{g_{ij}(s)}) - \eta_{z=0}(D_{g_{ij}(s)}), \quad (4.6)$$

with

$$g_{ij}(s) = \frac{N_{g_{ij}(s)}}{D_{g_{ij}(s)}}. \quad (4.7)$$

As a consequence:

$$n_{l_j} + \Psi_{z_i} \geq n_{z=0}(D_{g_{ij}(s)}) - n_{z=0}(N_{g_{ij}(s)}) \quad \forall j \in N. \quad (4.42)$$

Finally:

$$n_{l_i} \geq \max_{j \in N}(\eta_{z=0}(D_{g_{ij}(s)}) - \eta_{z=0}(N_{g_{ij}(s)})) - \Psi_{z_i} \tag{4.43}$$

5. Application

Let G_0 be the 2×2 transfer matrix:

$$G_0 = \begin{bmatrix} \dfrac{-0.5332}{(32s+1)^2(2s+1)} e^{-7.5s} & \dfrac{1.68}{(28s+1)^2(2s+1)} e^{-2s} \\ \dfrac{-1.2585}{(43.6s+1)(9s+1)} e^{-2.8s} & \dfrac{4.7861}{(48s+1)(5s+1)} e^{-1.15s} \end{bmatrix}. \tag{5.1}$$

A modified Alatiqi distillation column plant [18]. Some control design and process identification was applied to this process. Luyben in [18] proposed the so-called biggest log modulus tuning (BLT) method, Tavakoli in [19] proposed a decentralised PI controller to control the distillation column. In [20], Wang et al. proposed an identification process that leads to the following model:

$$G_0 = \begin{bmatrix} \dfrac{-0.5332}{67.7099s+1} e^{-19.5838s} & \dfrac{1.7171}{48.3651s+1} e^{-14.8791s} \\ \dfrac{-1.2585}{48.7805s+1} e^{-8.4505s} & \dfrac{4.7861}{49.7512s+1} e^{-4.9768s} \end{bmatrix}. \tag{5.2}$$

Plant (5.2) studied by Wang et al. in [20,21] illustrate the method proposed in this paper. In [22], Tao proposes an analytical decoupling control method based on the H_2 optimal performance objective of internal model control (IMC) theory. This method is compared to the proposed CRONE methodology. The CRONE methods provided better robust stability.

The control objectives are control the system and demonstrate the control system robust stability of the method, time-delays and time-constant uncertainties are ±20% of nominal values.

5.1. Determination of time-delay

With no approximation the determinant of $G_0(s)$ is

$$|G_0| = \frac{Ae^{-24.5606s} + Be^{-23.3296s}}{C}, \tag{5.3}$$

where

$$A = -2.5519(48.7805s + 1)(48.3651s + 1), \tag{5.4}$$

$$B = 2.1610(67.7099s + 1)(49.7512s + 1), \tag{5.5}$$

$$C = (67.7099s+1)(49.7512s+1)(48.7805s+1)(48.3651s+1). \tag{5.6}$$

Consequently, the time-delay of the determinant of $G_0(s)$ is $\alpha_0 = 23.3296$ and:

$$\tau(|G_0|) = 23.3296. \tag{5.7}$$

○ For the first loop:

$$\tau(G_0^{11}) = 4.9768, \tag{5.8}$$

$$\tau(G_0^{21}) = 8.4505, \tag{5.9}$$

also

$$\tau_1 = \min(\tau(G_0^{11}), \tau(G_0^{21})) = 4.9768, \tag{5.10}$$

and with Eq. (4.11):

$$\tau(\beta_{0_1}) \geq 23.3296 - 4.9768 = 18.3528. \tag{5.11}$$

○ For the second loop:

$$\tau(G_0^{12}) = 14.8791, \tag{5.12}$$

$$\tau(G_0^{22}) = 19.5838. \tag{5.13}$$

Consequently:

$$\tau_1 = \min(\tau(G_0^{12}), \tau(G_0^{22})) = 14.8791, \tag{5.14}$$

and by relation (4.11):

$$\tau(\beta_{0_2}) \geq 23.3296 - 14.8791 = 8.4505. \tag{5.15}$$

5.2. Determination of the positive real part zeros and poles

First to determine the various poles or zeros of G_0^{-1} that must be appear on each nominal open-loop transfer function $\beta_{0_i}(s)$, the zeros and poles of $\left|G_0\right|_{\alpha_0}$ and $\left|G_0^{ij}\right|_{\delta_{0_{ij}}}$ must be calculated. The Padé approximation is used. The first order Padé approximation of a L_{ij} time-delay is

$$e^{-L_{ij}s} = \frac{1 - (L_{ij}/2)s}{1 + (L_{ij}/2)s}. \tag{5.16}$$

For all time-delays the approximation has been calculated. A Matlab function shows that the dominant zero of the determinant of $\left|G_0\right|_{\alpha_0}$ is $z = 0.0129$:

$$\eta_z\left(\left|G_0\right|_{\alpha_0}\right)_{z=0.0129} = 1. \tag{5.17}$$

The plant has no zeros and poles in the right half-plane. Consequently:

$$\begin{aligned} &\eta_z\left(\left|G_0^{11}\right|_{\delta_{0_{11}}}\right) = 0 \\ &\eta_z\left(\left|G_0^{12}\right|_{\delta_{0_{12}}}\right) = 0 \\ &\eta_z\left(\left|G_0^{21}\right|_{\delta_{0_{21}}}\right) = 0 \\ &\eta_z\left(\left|G_0^{22}\right|_{\delta_{0_{22}}}\right) = 0 \end{aligned} \quad \forall z \in \mathbb{C}^+, \tag{5.18}$$

Also:

$$\eta_1(z) = \min_{z=0.0129}\left(\eta_z\left(\left|G_0^{11}\right|_{\delta_{0_{11}}}\right), \eta_z\left(\left|G_0^{21}\right|_{\delta_{0_{12}}}\right)\right) = 0, \tag{5.19}$$

and

$$\eta_2(z) = \min_{z=0.0129}\left(\eta_z\left(\left|G_0^{12}\right|_{\delta_{0_{21}}}\right), \eta_z\left(\left|G_0^{22}\right|_{\delta_{0_{22}}}\right)\right) = 0, \tag{5.20}$$

Finally by Eq. (4.24):

$$\eta_z((\beta_{0_i})_{0_1})|_{z=0.0129} = 1 - 0 \geq 1, \tag{5.21}$$

and

$$\eta_z((\beta_{0_i})_{0_2})|_{z=0.0129} = 1 - 0 \geq 1. \tag{5.22}$$

Consequently, the two transfer functions of the nominal open-loop transfer function matrix contain a RHP zero at $z = 0.0129$.

Finally:

$$\beta_{0_1} = C_{z_1}\beta_{l_1}(s)(s - z_1)\frac{n_1}{d_1}e^{-18.3528s}\beta_{h_1}(s), \tag{5.23}$$

and

$$\beta_{0_2} = C_{z_2}\beta_{l_2}(s)(s - z_2)\frac{n_2}{d_2}e^{-8.4505s}\beta_{h_2}(s), \tag{5.24}$$

where $z_1 = z_2 = 0.0129$.

Then $n_1, d_1, n_2, d_2, C_{z_1}$ and C_{z_2} are optimized.

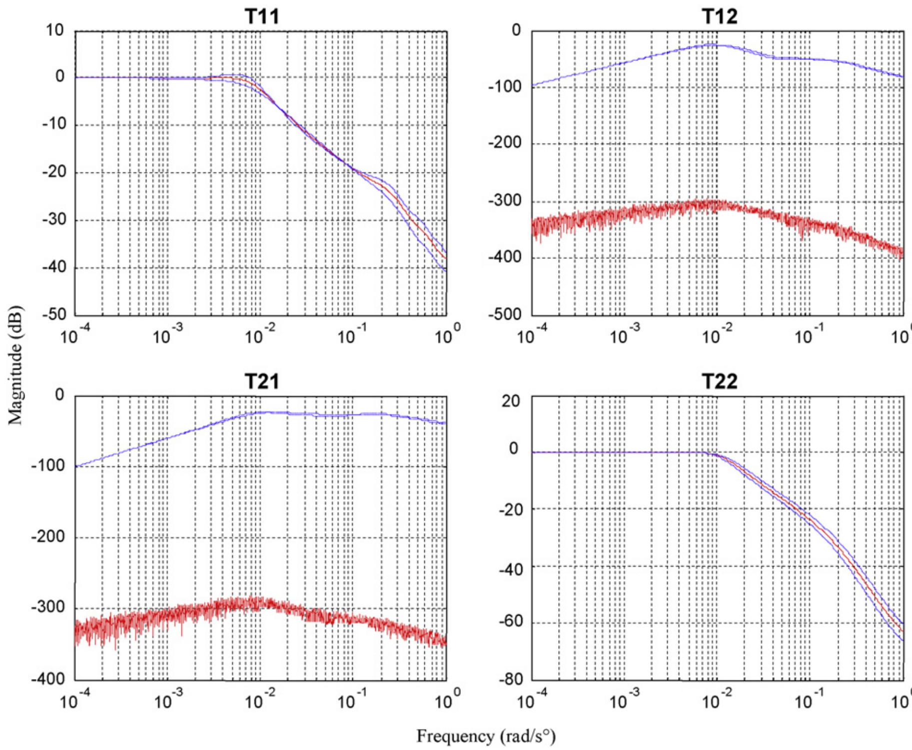

Fig. 3. (–) Nominal and (–) reparametered frequency responses of fractional complementary sensitivity.

5.3. Specifications and optimization results

5.3.1. Specifications

For all the possible parametric states, the control must satisfy the following performance specification:

- a zero steady-state error for the two outputs,
- a perfect decoupling for the two loops,
- a smaller as possible settling time,
- a first overshoot less than 10%.

Eq. (4.43) and first specification imposes $n_l \geq 1$. The second specification imposes finite gain to be smaller as possible, for the non-diagonal transfer function of the complementary sensitivity function. The last specification will be satisfied by a nominal resonant peak equal to 10^{-3} for the two loops.

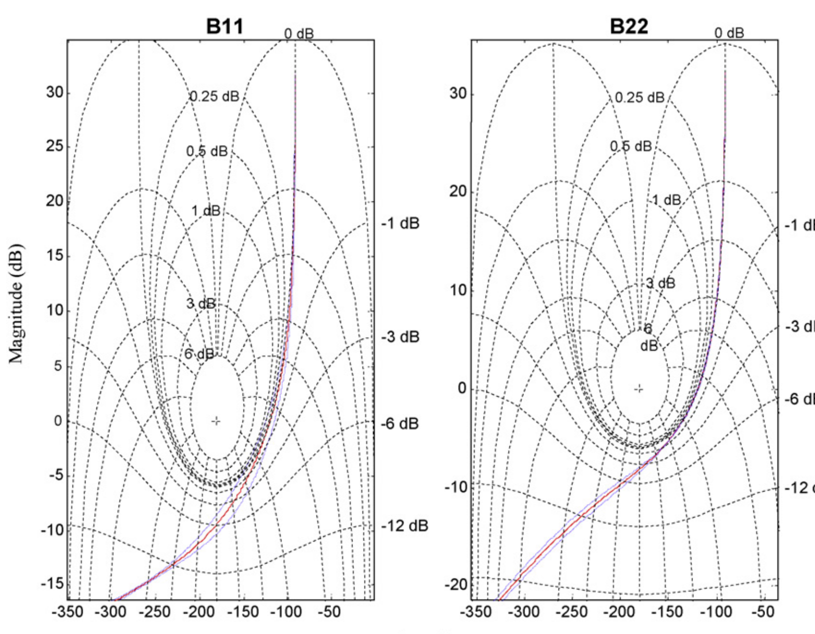

Fig. 4. (–) Nominal and (–) reparametered fractional open-loop frequency responses.

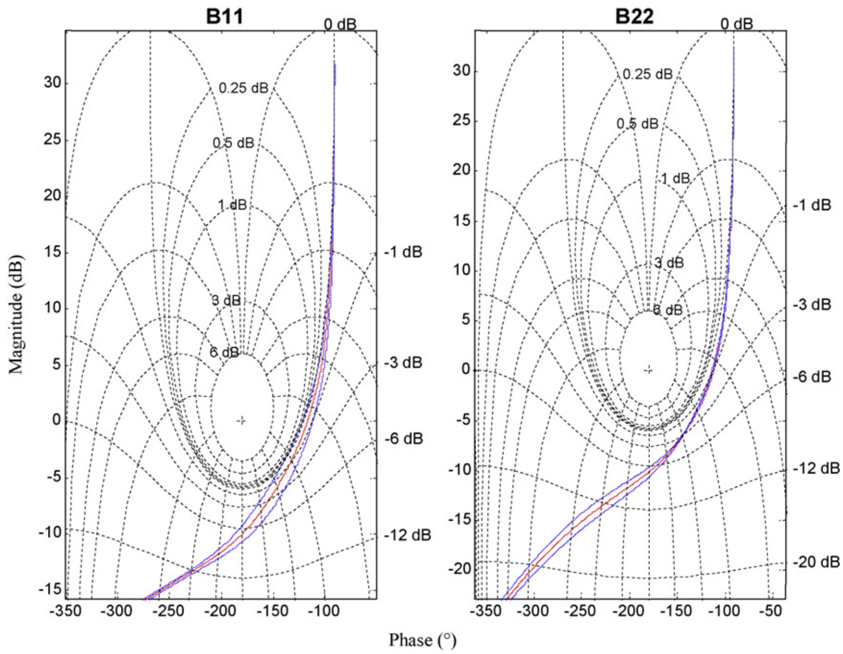

Fig. 5. (–) Nominal and (–) reparametered rational open-loops using the first controller design.

The inputs of the CRONE control optimization software are:

- the various RHP zeros and unstable poles of $\beta_{0_i}(s)$,
- the time-delay for each control loop,
- an initialisation of the various elements of the non-integer open-loop transfer function: ω_r, ω_h, ω_l, $||\beta_{0_i}(\mathrm{j}\omega)||_{\omega=\omega r}$, n_l and n_h.

For the first loop, the initial values are:

- $\omega_r = 0.0042$ rad/s,
- $\omega_h = 0.4$ rad/s,
- $\omega_l = 0.002$ rad/s,
- $||\beta_{0_1}(\mathrm{j}\omega)||_{\omega=\omega r} = 0$ dB,

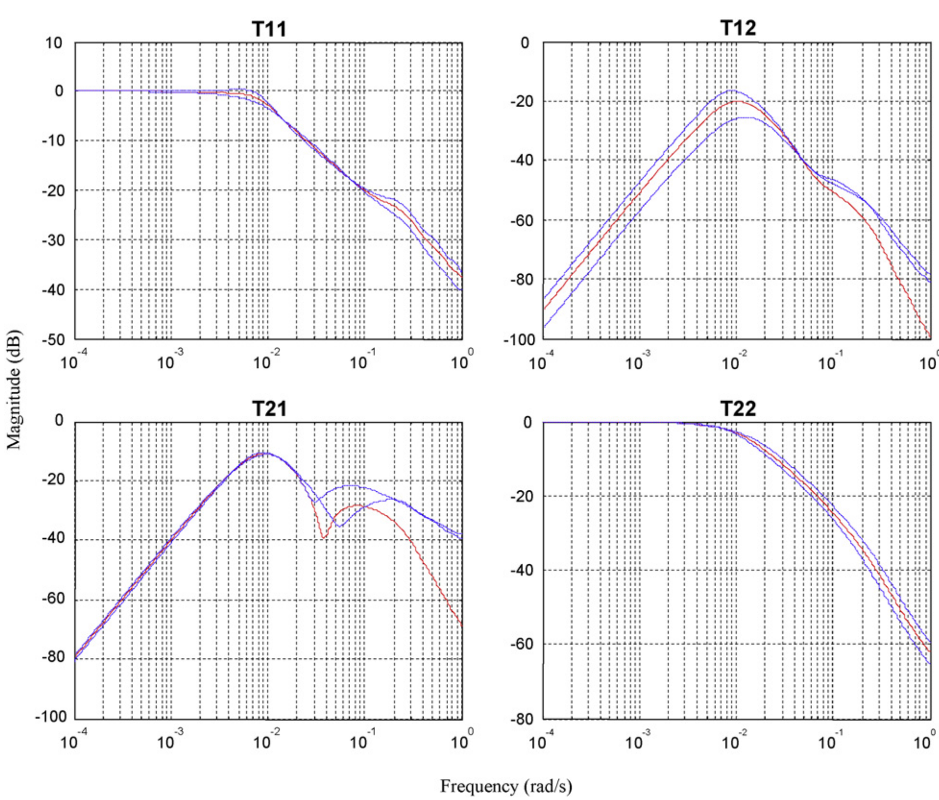

Fig. 6. (–) Nominal and (–) reparametered complementary sensitivity functions for the rational controller using the first controller design.

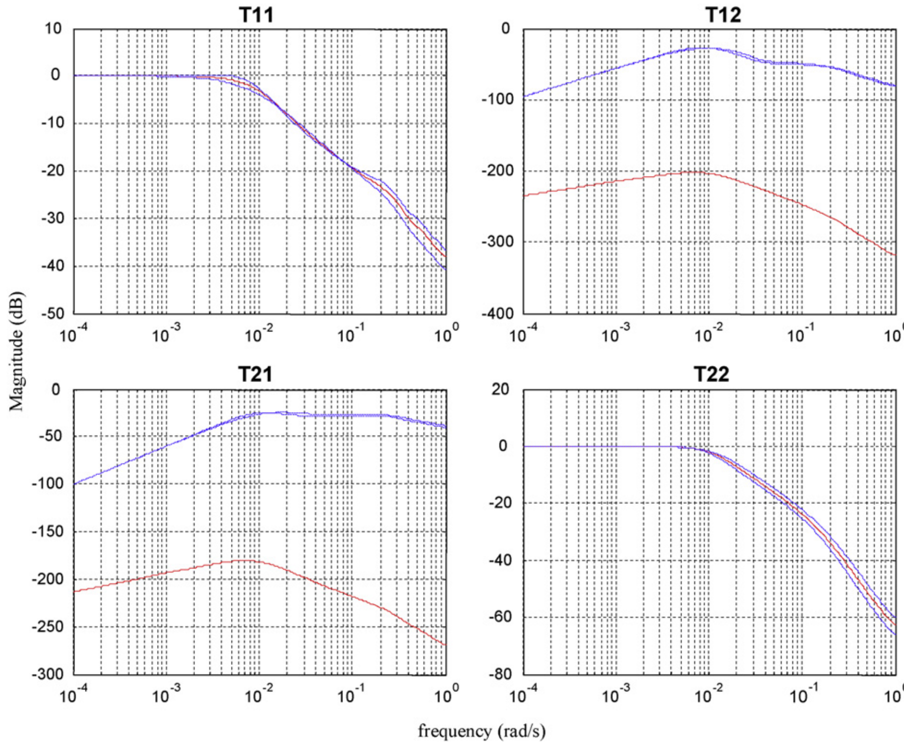

Fig. 7. (–) Nominal and (–) reparametered complementary sensitivity functions for the rational controller using the second controller design.

- $n_l = 1$,
- $n_h = 2$ for the controller be proper (4.37).

And for the second loop, the initial values are:

- $\omega_r = 0.0041$ rad/s,
- $\omega_h = 0.4$ rad/s,
- $\omega_b = 0.01$ rad/s,
- $\left\|\beta_{0_2}(\mathrm{j}\omega)\right\|_{\omega=\omega_r} = 0.5\,\mathrm{dB}$,
- $n_l = 1$,
- $n_h = 2$ for the controller to be proper (4.37).

Consequently, the open-loop transfer function matrix to optimize is

$$\beta_0(s) = \begin{bmatrix} \beta_{0_1}(s) & 0 \\ 0 & \beta_{0_2}(s) \end{bmatrix}. \qquad (5.25)$$

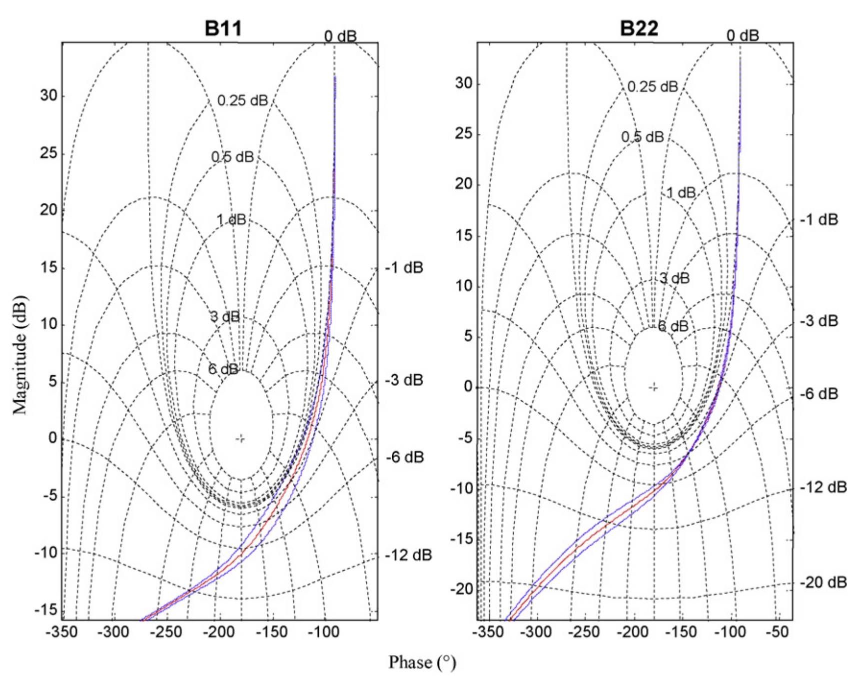

Fig. 8. (–) Nominal and (–) reparametered rational open-loop transfer functions using the second controller design.

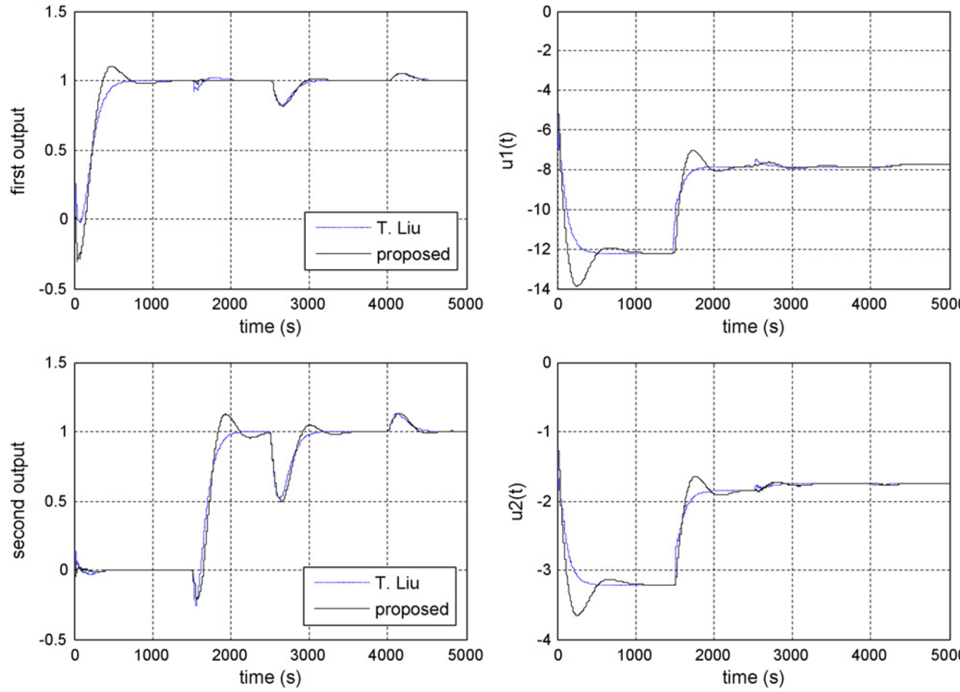

Fig. 9. Nominal response (left outputs, right control signal).

In which:

$$-\beta_{0_1}(s)=\beta_{l_1}(s)\beta_{0_1}(s)\beta_{h_1}(s), \tag{5.26}$$

$$\beta_{0_1}(s)=(s-0.0129)C_{11}^{\mathrm{sign}(b)}\left(\frac{1+s/0.4}{1+s/0.002}\right)^{a_1}\times\left(\Re e_{/\mathrm{i}}\left\{\left(C_{g_1}\frac{1+s/0.4}{1+s/0.002}\right)^{\mathrm{i}b_1}\right\}\right)^{-q_1\,\mathrm{sign}(b_1)}e^{-18.3528s}, \tag{5.27}$$

$$\beta_{l_1}(s)=C_{l_1}\left(\frac{0.002}{s}+1\right), \tag{5.28}$$

and

$$\beta_{h_1}(s)=\frac{C_{h_1}}{((s/0.4)+1)^2}. \tag{5.29}$$

$$-\beta_{0_2}(s)=\beta_{l_2}(s)\beta_{0_2}(s)\beta_{h_2}(s), \tag{5.30}$$

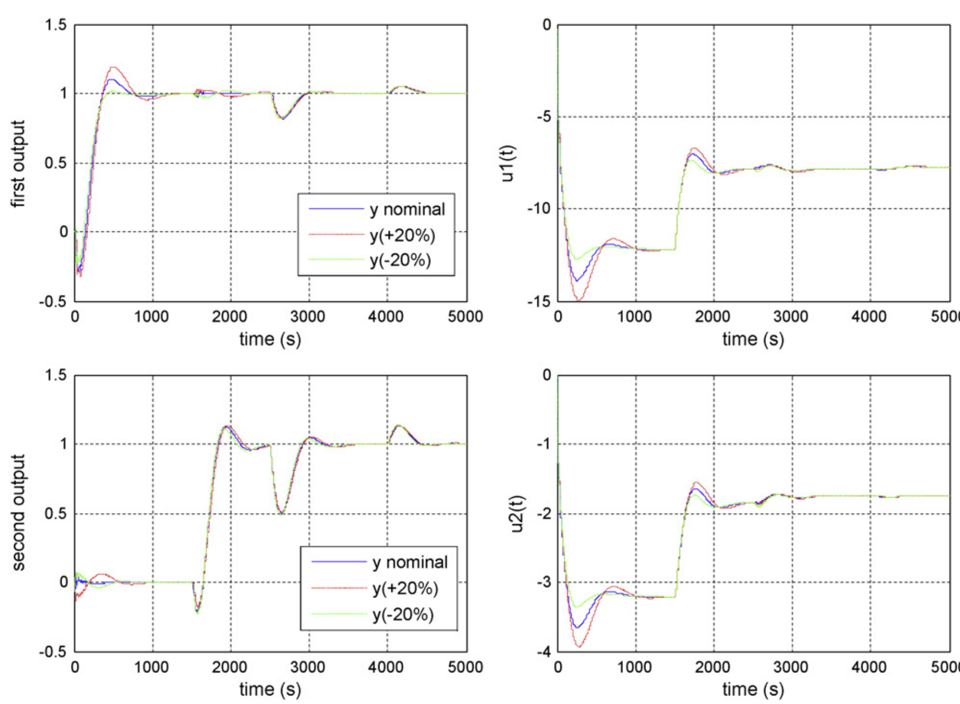

Fig. 10. Perturbed system response (left outputs, right control signal).

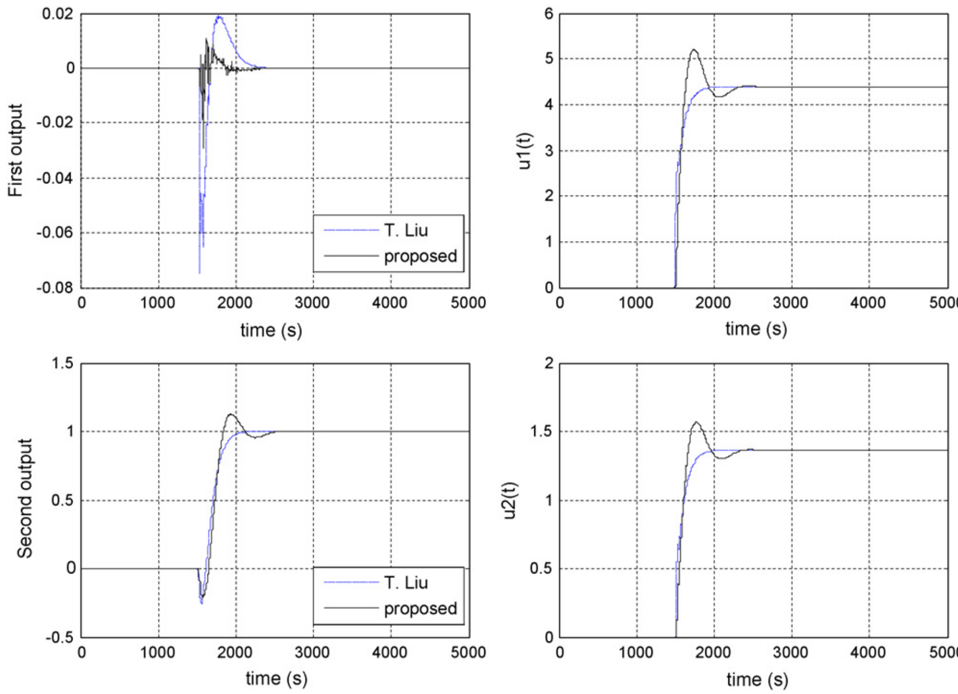

Fig. 11. First output response to a unit step in the second input.

$$\beta_{0_2}(s) = (s - 0.0129)C_{22}^{\mathrm{sign}(b)}\left(\frac{1+s/0.4}{1+s/0.01}\right)^{a_2} \times \left(\Re e_{/\mathrm{i}}\left\{\left(C_{g2}\frac{1+s/0.4}{1+s/0.01}\right)^{\mathrm{i}b_2}\right\}\right)^{-q_2\mathrm{sign}(b_2)} e^{-8.4505s}, \quad (5.31)$$

$$\beta_{l_2}(s) = C_{l_2}\left(\frac{0.01}{s}+1\right), \quad (5.32)$$

and

$$\beta_{h_2}(s) = \frac{C_{h_2}}{((s/0.4)+1)^2}. \quad (5.33)$$

Optimization is infact consist in finding the optimal value for $K_i (K_i = C_{ii}^{sign(b)} C_{h_i} C_{l_i})$, a_i, b_i, q_i and C_i.

5.3.2. Results

Taking into account all the specifications, the optimal values for the various parameters are:

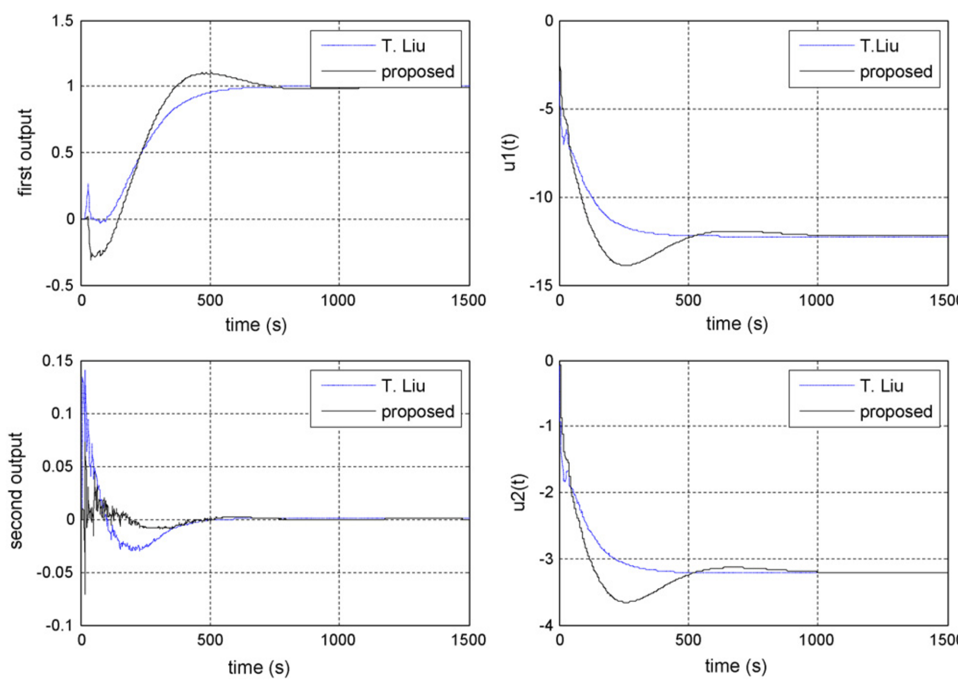

Fig. 12. Second output response to a unit step in the first input.

- for the first loop: $K_1 = 2.0160$, $a_1 = 1.1087$, $b_1 = -0.3710$, $q_1 = 2$ and $C_1 = 2.3258$.
- for the second loop: $K_2 = 0.4122$, $a_2 = 1.3668$, $b_2 = -0.6808$, $q_2 = 4$ and $C_2 = 1.0807$.

For the ideal fractional open-loop, all specifications have now been verified. In fact Fig. 3 shows that magnitudes, for all diagonal elements (nominal or reparametered) are around zero. However, for non-diagonal elements there is a distinction between nominal and reparametered elements. The magnitude for the nominal plant is smaller (around −300 dB) but for the reparametered plants it is around −25 dB. Also Fig. 4 shows that the frequency responses of the ideal fractional open-loop overlap the 0 dB M-contour.

5.4. *The controller*

First, for find the controller frequency response, frequency-domain and $K = G_0^{-1}\beta_0$ are used. To the frequency response of the fractional controller, poles and zeros are used to synthesize a rational controller.

The expression of the controller is

$$^{10}k_{ij}(s) = \frac{G_0^{ji}(s)}{|G_0(s)|}\beta_{0_i}(s) \quad \forall i, j \in N, \tag{5.34}$$

and the expression of the inverse of the determinant is

$$\frac{1}{|G|} = \frac{C}{Ae^{-24.5606s} + Be^{-23.3296s}}. \tag{5.35}$$

To *synthesize* the rational *controller two methods* can be used.

5.4.1. *Controller design: first method*

This first method consists in not used a priori known elements of the transfer G_0^{-1} when frequency-domain system identification is used to find the rational controller. The parameters of the reduced-order transfer function with a predefined structure are tuned to fit the fractional frequency response of the ideal controller. The rational integer model on which the reduced-order transfer function is based, is given by

$$k_{ij}(s) = \frac{A(s)}{B(s)}. \tag{5.36}$$

where $A(s)$ and $B(s)$ are polynomials of specified integer degrees n_a and n_b. An advantage of this design method is that whatever the complexity of the control problem, around six satisfactory values of n_a and n_b are easily found. In our case this simplicity is detrimental to performance. This controller synthesis method gives non-optimal results. Even if the equivalent open-loop overlaps the 0 dB M-contour (Fig. 6), the second specification is not respected. When comparing the ideal closed-loop (Fig. 3) and the rational closed-loop (Fig. 5) a perfect decoupling for two loops is not achieved. In fact the non-diagonal elements show, for the nominal plant, a maximum attenuation of 20 dB for the first loop and around 10 dB for the second.

5.4.2. *Controller design: second method*

In the second design an a priori known is used when the final and rational controller is searched. In this case many known elements are introduced in the initial and predefined structure of the transfer functions. In (5.33) and (5.34), it can be seen that the $G_0^{ji}C$ elements for each element of K are not necessary for approximation.

In our case this method gives optimal results. The ideal fractional closed-loop (Fig. 3) or open-loop (Fig. 4) is match with the rational closed-loop (Fig. 7) or open-loop (Fig. 8).

Table 1
Controller transfer matrix coefficients.

	k_{11}		k_{12}	
	a_i	b_i	a_i	b_i
1	1.570	0	1.570	0
2	0.748	0.522	0.835	0.458
3	0.042	0.167	0.835	0.271
4	0.021	0.095	0.498	0.167
5	0.020	0.023	0.020	0.118
6	0.015	0.021	0.020	0.032
7			0.015	0.021
	k_{21}		k_{22}	
	a_i	b_i	a_i	b_i
1	1.570	0	1.570	0
2	0.748	0.522	0.835	0.458
3	0.042	0.167	0.835	0.271
4	0.021	0.095	0.498	0.167
5	0.020	0.023	0.021	0.118
6	0.015	0.021	0.020	0.032
7			0.020	0.021

The second design gives the best results. Each element of the controller transfer matrix is based on the model defined by (5.35) (Table 1).

5.5. *Assessment of the controller*

A simulation is now carried out to assess the controller.

Step signals are applied, respectively, at reference inputs at $t = 0$ s and $t = 1500$ s, and at input disturbances at $t = 4000$ s and $t = 2500$ s. Their magnitudes are, respectively, 1 and 0.1. Fig. 9 presents the nominal plant inputs and outputs. Fig. 10 presents the perturbed and nominal plant inputs and outputs. As the outputs always follow the reference inputs without poor damping (robust stability degree), the controller can be said robust. It is also efficient as it always rejects the disturbance effect. Fig. 11 and 12 show that the controller designed using the CRONE approach (black line) is more decoupling than the controller designed using T. Liu approach (dotted line).

6. Conclusion

The CRONE methodology has been extended to unstable multivariable plant with RHP zeros and multiple time-delays. To treat this kind of plant some elements of G_0 or its inverse must be introduced in each open-loop, this article gives a whole method to find it.

A chemical engineering process is used to improve the proposed approach. The extended CRONE control approach has been used to design a controller to ensure robust closed-loop stability degree, decoupling and disturbances rejections. The simulation exerted show that CRONE control approach successfully states the robust stability of the closed-loops, the robust decoupling and the robust disturbances rejection.

A crone approach to unstable and non-square multivariable plants with multivariable time-delay, and RHP zeros, is being investigated.

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